DEAF STUDENTS SOLVING OF ARITHMETIC WORD PROBLEMS

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There has been little research into the intersection of language and arithmetic performance of deaf students although previous research has shown that deaf and hearing-impaired students are delayed in their language acquisition and arithmetic performance. This paper examines the performance of deaf and hearing-impaired students in South-East Queensland in solving arithmetic word problems.

Although the poor performance of deaf student in comparison with their hearing peers has been documented in a number of achievement areas, particularly literacy (see (Hine, 1980; Wilson & Hyde, 1997) not a great deal is known about the mathematical knowledge and related conceptual development of deaf students (Wood, Wood Griffith & Howarth, 1986; Titus, 1995). Research examining deaf students' achievement in mathematics has chiefly concentrated on the study of their skills in operations and number. These studies have generally concluded that there is no significant cognitive basis for major differences in mathematical performance between deaf and hearing student and that achievement differences that are observed are the result of a combination of linguistic and experiential delays for the deaf students (Titus, 1995; Serrano Pau, 1995). Increasingly, the role of language in mathematic comprehension and application to problem solving is recognised – for both hearing and hearing impaired/deaf students.

Stone (1980) reported the difficulty that deaf and hearing-impaired students had when sequencing. The capacity to sequence requires students to place things in order so that there is a need for understanding "greater than" and "less than", but it was found that hearing-impaired students undertaking O levels (16+) and A levels (18+) experienced difficulty in writing things in order in the science laboratory and in following the steps of theorem. The apparent delay in critical components of mathematical thinking such as sequencing and conservation are not due to inferior cognitive abilities but rather a lack of experience and language (Rittenhouse & Kenyon, 1991).

Early studies (Furth, 1966) concluded that while deaf students' limited success with mathematics demonstrated that they were capable of logical thought, they evidenced slower rates of educational development when compared to their hearing peers. Hine (1970) in a large scale standardised test found that hearing-impaired students of 10 years were working at the level of 8.5 year old peers in mathematics and that by the age of 15, they were working at the level of 10.5 year olds in mechanical mathematics and 11 year olds in problem arithmetic. Suppes (1974) made similar claims, but with recognition that cognitive performance was comparable with hearing students when the task did not involve verbal skills. More recently, researchers agree that the most difficult problems for deaf students in mathematics are those problems that involve linguistic applications, again indicating that language is a key to mathematical performance.

Barham and Bishop (1991) have noted that the linguistic complexity of mathematical terminology which can be confusing for deaf students. Terms such as "tens" and "tenths" or "sixty" and "sixteen" are difficult for students using lip reading as they look very similar on the lips. Other difficulties can be centred on the unique ways in which words are used in mathematics which are very different from their non-school use. For example, the use of "up" in the non-school context and then "add up" in the mathematics classroom can be very confusing. Kidd and Lamb (1993) also comment on words with double meaning such as "interest" and "table". Such multiplicity of meaning is very confusing for deaf students. Furthermore, they argue that the technical vocabulary (such as "reciprocal", "premium") along with variations of a form ("year/yearly", "base/basic") can result in confusion. There are also numerous ways of expressing the same idea or concept so for deaf students where there may difficulty in understanding a concept one way, there is greater difficulty when the same concept is expressed in a different manner. Another area of difficulty is with logical connectives such as "if" and "because". These are rarely explicitly taught to hearing students, but rather they learn them through interactions. In contrast, hearing-impaired students do not have much opportunity to learn the meaning of such words, yet they are essential elements for and used in much of mathematics.

Extending beyond word structures, to more complex configurations, word problems of an everyday nature involving the use of linguistic forms applied to arithmetic concepts and strategies have been found to cause difficulties (Daniele, 1993; Serrano Pau, 1995; Wood et al., 1986). The exact relationship between language and mathematics for deaf students has not been clearly established with some researchers indicating that there is some relationship with the levels of literacy in English language instruction (Zwiebel & Allen, 1988), while other suggest it is related to the level of hearing loss (Wood et al., 1986).

Serrano Pau (1995) studied deaf students attempting change, compare and combine problems in order to examine their problem solving abilities in comparison with their reading comprehension levels. While he acknowledged that the relationship between reading comprehension and problem solving ability was important, he acknowledged that the ineffective problem solving strategies adopted by the students, which were largely based upon on the strategies taught by teachers, were also relevant in the students' poor performance. Luckner and McNeill (1994) extend this research on linguistic competence and propose three learner capabilities were relevant for deaf students:

a) intellectual skills – including a knowledge of the concepts and rule structures relating to the problem solving task;
b) organised information in the form of appropriate schemata to enable an understanding of the problem; and
c) cognitive strategies that allow the learner to select the relevant information and strategies necessary to the problem's solution.

Luckner and McNeill (1994) suggested that deaf students may have difficulties in relation to arithmetic problem solving and it was important to identify ways to assist deaf students to develop organisational and procedural skills if their arithmetic solving were to be improved.

In summary, researchers have claimed that while deaf students consistently show delays in comparison with their hearing peers in arithmetic problem-solving, there is no simple or direct relationship established between these delays and the students' linguistic and experiential deficits or their degree of hearing loss (Wood, et al, 1986). In general, teacher ratings have been demonstrated to be effective predicators of student communication and academic performance (Hyde & Power, 1996). While only a small number of researchers have examined deaf students performance on problem solving tasks, even fewer studies have considered in detail their performance on arithmetic word problems.

This research reports on a project investigating Queensland deaf students' performance on arithmetic word problems. As previous studies have not considered the strategies used by deaf students in solving arithmetic word problems, this project investigated the following questions:

1. How do deaf and hearing-impaired students compare with their hearing peers when solving arithmetic word problems?
2. Are there identifiable strategies used by deaf and hearing-impaired students when solving arithmetic word problems?

Using the research instrument developed by Lean, Clements and Del Campo (1990) and their data as a comparison base, this project investigated the outcomes and strategies used by Queensland deaf and hearing-impaired students when solving arithmetic word problems.

METHOD

Sample: All moderately through to profoundly deaf students in South-East Queensland were surveyed and a sample were interviewed. Students from both mainstream and Special Education settings were included in the study. The students had been ascertained to require special educational assistance because of their hearing loss according to Education Queensland Guidelines for Ascertainment of Hearing-Impaired students. All students had mastered basic number facts (ie in this context, basic addition and subtraction facts to 100) and had basic English competency skills as determined from school records and teacher judgements. Comparable numbers of boys and girls participated in the study and students did not other significant or uncorrected impairments. In some cases, the sample size for a particular grade level is low due to the number of students enrolled in any particular year level.

Table 1
Number of student from each grade level.

Gr	1	2	3	4	5	6	7	8	9	10	11	12
N	3	6	7	4	11	12	6	15	7	3	0	3

The Task: The arithmetic word problems were those categorised by Heller and Greeno (1978) and used in the Australian and Papua New Guinea (PNG) context with hearing students by (Lean et al., 1990). The three types of questions – “change”, “combine” and “compare” – were used in the study. All questions involve additive or subtractive strategies with numbers where the sum is less than 10.

Method: The study involved two key phases. Phase One consisted of a survey instrument which students across South-East Queensland completed. The survey consisted of the 24 word problems developed by Lean et al (1990) and using a pencil-and-paper format. The implementation of the survey was supervised by a trained research assistant (mathematics and hearing-impaired trained teacher) and in conjunction with the classroom teacher. This phase provided base-line data to compare deaf and hearing impaired students' performance against that of the Lean et al study. From the data collected through the survey, target students were identified and up to 6 students each grade level were interviewed to provide more in-depth data on the strategies used to solve the word problems. Phase Two consisted of the development of case studies of the target students. Students were encouraged to use a “think aloud” strategy to solve the tasks. The interviews were video-recorded and were undertaken with a trained sign-language research assistant. Students were able to use their preference for signed English or oral communication to present their responses to the tasks. All interviews were transcribed and analysis undertaken of both the mathematical and non-mathematical language used in the task resolution. The responses offered provided indicators of the problem-solving strategies used, features of the language used, of cognitive strategies and of mathematical reasoning.

Results

Table 2
Percentage of correct responses for each item

		1	2	3	4	5	6	7	8	9	10	12	tot
1	Ann has 2 cents. Mary has 3 cents. Who has more cents, Ann or Mary?	100	100	100	75	100	100	83.3	93.3	100	100	100	96.1
2	Bill has 5 cars. Mark has 2 cars. Who has less cars, Bill or Mark?	66.6	83.3	85.7	75	90.9	91.6	66.6	86.6	85.6	100	100	85.7
3	Which number is more, 4 or 7?	66.6	100	100	100	100	100	100	93.3	100	100	100	97.4
4	Which number is less, 3 or 5?	33.3	100	85.7	75	90.9	83.3	100	100	100	100	100	90.9
5	Tania had 3 apples. Ben had twice as many as Tania. How many apple did Ben have?	0	0	14.3	0	9.1	25	33.3	60	42.9	100	66.6	31.2
6	My jug holds 6 litres of milk. It holds twice as much as Luke's jug. How much milk does Luke's jug hold?	0	0	14.3	0	0	0	16.6	0	0	100	33.3	7.8
7	Which number is 3 more than 4?	0	0	14.3	0	18.2	8.3	33.3	53.3	28.6	100	33.3	26
8	Which number is 2 less than 3?	0	0	28.6	0	18.2	25	66.6	66.6	28.6	100	66.6	36.4
9	5 is 1 more than which number?	0	16.6	0	0	18.2	0	16.6	20	14.3	66.6	33.3	14.3
10	6 is 2 less than which number?	0	16.6	14.3	0	0	0	0	6.7	0	66.6	33.3	7.8
11	Barbara had 2 eggs. Dan gave Barbara 1 more egg. How many eggs does Barbara have now?	66.6	66.6	85.7	75	100	66.6	100	93.3	85.7	100	66.6	84.4
12	Jack has 4 pens. Dianne took 3 of Jack pens. How many pens did Jack have then?	33.3	83.3	85.7	75	90.9	66.6	100	80	100	100	66.6	81.8
13	Jeff had 3 bananas. Carmel gave Jeff some more bananas. Jeff then had 5 bananas. How many bananas did Carmel give Jeff?	33.3	16.6	42.9	25	18.2	41.6	50	53.3	85.7	100	66.6	45.5
14	Anna had five books. Tom took some of Anna's books. Then Anna only had 2 books left. How many of Anna's books did Tom take?	33.3	50	71.4	75	54.5	66.6	66.6	80	100	100	66.6	70.1
15	Paul had some pencils. His father gave him 2 more pencils. Then he had 5 pencils. How many pencils did Paul have at the start?	0	0	28.6	25	36.4	58.3	50	60	85.7	100	66.6	48.1

		1	2	3	4	5	6	7	8	9	10	12	tot
16	Sally has some pictures. She lost 2 of her pictures. Then she had 3 pictures. How many pictures did she have at the start?	0	0	42.9	0	36.4	50	50	80	57.1	100	33.3	46.8
17	David has 2 dogs and Jim has 4 dogs. How many dogs do they have altogether?	66.6	83.3	100	100	90.9	100	100	100	100	100	100	96.1
18	Helen has 3 ribbons. Lyn also has some ribbons. Helen and Lyn have 7 ribbons altogether. How many ribbons does Lyn have?	0	0	0	25	27.3	25	66.6	66.6	85.7	100	66.6	41.6
19	John has 2 buckets. Eric has 6 buckets. How many more buckets than John does Eric have?	0	0	0	0	18.2	8.3	33.3	60	57.1	66.6	66.6	28.6
20	Nick has 2 cups. Sarah has 7 cups. How many cups less than Sarah does Nick have?	0	0	0	0	27.3	33.3	33.3	86.6	71.4	100	66.6	41.6
21	Gina has some boxes. Ken has 3 boxes. Gina has 1 more box than Ken. How many boxes does Gina have?	0	50	85.7	50	54.5	83.3	83.3	93.3	71.4	100	66.6	72.3
22	Jo has some dolls. Pat has 5 dolls. Jo has 2 dolls less than Pat. How many dolls does Jo have?	0	0	0	50	36.4	16.6	33.3	86.6	57.1	100	66.6	41.6
23	Bill has some trucks. Tina has 5 trucks. Tina has 2 trucks more than Bill. How many trucks does Bill have?	0	16.6	0	25	45.5	25	50	33.3	28.6	100	66.6	32.5
24	Sam has some marbles. Sarah has 6 marbles. Sarah has 2 marbles less than Sam. How many marbles does Sam have?	0	33.3	71.4	0	0	50	33.3	40	14.3	100	33.3	33.8

The overall could not be analysed for statistical significance due to the small numbers in each year level cohort. However, the trends confirm similar patterns of difficulty experienced for hearing and non-hearing when comparison is made with the Lean et al (1990) data. As noted by other authors researching aspects of development of numeracy, mathematical and arithmetic concepts for deaf and hearing-impaired students, the data in this project further confirms the delay in the capacity of deaf and hearing-impaired to respond appropriately to tasks such as those commonly found in mathematics classrooms.

DISCUSSION

For many students – hearing and hearing-impaired – there is a reliance on trigger words which offer students cues as to the meaning of the problem. For example, the trigger word "more" is often seen to represent an additive operation. In part, this is potentially due to the widespread practice of teachers to introduce the addition operation in the context of word problems relevant to the students' non-school experiences. Common practices consist of teachers posing questions such as "I had three apples, then I bought three more, how many did I have altogether?" Such tasks expose students to practices where they can extrapolate the meaning of "more" to imply addition. For students whose access to the lexical density and complexity of the mathematics register, such as deaf students, their capacity to decipher the very subtle use of language is restricted.

Two trigger words which appeared to be particularly difficult for deaf students were "twice" and "less". Items 5 and 6 used the word "twice" in the context of doubling and halving respectively. For many students below Grade 6, the term itself was difficult with a number of the students asking for clarification of meaning. Where those students were able to offer a correct response, only one student was able to justify the response thus raising questions as to the students' deep understanding of the concept. For some students who were able to articulate their reasoning, "twice" meant "double". In most cases, such a crude understanding would enable them to solve many word problems. However as was apparent in Item 6 where the inverse was the case, students needed to halve the amount in order to respond correctly. As indicated by the overall performance, this task appeared to be one of the most difficult for the students. Most frequently, students relied on their understanding of "twice" to mean "double" and would offer a response of 12. Unlike the other tasks where the problems require the manipulation of different sets, this task required a doubling (or halving) operation and it would appear that the complexity of the task is in the specificity of the operation/s associated with the use of "twice".

Other common trigger words in mathematics are the terms "more" and "less" which are often associated with the operations of addition and subtraction respectively. The patterns of responses confirmed that these patterns. Furthermore, as Walkerdine (1982) has noted, for working-class students, whose linguistic code is somewhat restricted in comparison with that of the middle-class and formal school codes, deaf students found the concept of "less" particularly difficult. At the decontextualised level (Items 3 and 4), where there the word problems are not embedded within a context, where the binary oppositional terms of "more" and "less" were used, there were fewer students able to respond correctly to Item 4 where the relational term "less" was integral to the item. Further through the tasks, students misconstrued items using the term "less" in a number of different ways. In the comparative tasks such as Item 8 where the task asks which number is 2 less than 3, some students perceived the use of "less" to connote subtraction and were able to respond correctly, in spite of not understanding the task. However, most students saw the task as a comparative one in which they had to identify which was the lesser in value of 2 or 3. The wording of Items 7 and 8 were interpreted similarly, whereby the students saw them as asking which was the greater (or lesser) of the two numbers, rather than as addition or subtraction equations. The signifier "than" seemed to be redundant wording as many of the students did not know the word nor its role in the sentence, resulting in the (mis)interpretation of the task as being "which is more, 3 or 4".

The use of prepositions has been found to cause difficulties for students whose first language is not English. McGregor (1991), working in the multicultural classroom, found that non-English speaking background students found the use of propositions particularly difficult. For example, she cites the example where the statements are made: The temperature fell by 10 degrees ....to 10 degrees, ...from 10 degrees, ... Rudner (1978, cited in Barhnam & Bishop,. 1991, p. 182)) noted a similar difficulty with deaf students who struggled with conditionals (if, when); comparatives (greater than, less than), negatives (without, not) and inferentials (should, could). Difficulties were noted with the use of "is" in Items 9 and 10 where interpreted the question as being 5 and 1 more is which number (as opposed to 5 is 1 more than which number).

Contextualised Word Problems

Where the problems are stated simply and in order of operation, there was a greater likelihood of the students being able to offer correct responses (such as Items 11 and 12). The actions involved in the change tasks (giving and taking) provided strong contextual cues for the students as to the appropriate operation. Difficulty arose when the indefinite "some" came into the word problems. Not only was the amount missing but it also produced confusion as to the appropriate action (operation) to be undertaken. Where the non-stated amount came at the first line in the task, the students experienced great difficulty in making sense of the task. In Items 15 and 16, the lack of a definitive amount confused the students as to the requirements of the task (find out how many were there at the beginning). The students searched for key words (such as "lost") to offer some cues as to what they might need to do. In the case of Item 16 where something was "lost" students took this as a cue for subtraction. Where the first line is ambiguous, the students ignored it and saw it as having no relevance to the task. They then proceeded to operate on the number given using what cues they could extract from in the task.

The comparative tasks in Items 19 and 20 proved to be difficult due to the structure of the final sentences, which required a comparison to be made and a difference noted. As with the decontextualised tasks, students were at risk of interpreting the task as one of which was the bigger or smaller quantity and naming that quantity. Alternatively, they would interpret the "more" or "less" in the wording to signify addition or subtraction.

CONCLUSION

The data collected confirmed previous studies where deaf students are somewhat delayed in their performance in mathematics. Furthermore, the data collected in this project support the notion that the specificity of language used in mathematics – or more specifically-arithmetic word problems, creates difficulties for deaf students.

Many of the deaf students relied on top-down approaches to comprehending the tasks, recognising that they would not understand everything that they had read. Relying on this "key word" reading strategy, they use their understanding of key words and this knowledge base to construct a meaning for the tasks. Such strategies often mean that signifiers such as "more" and "less" are interpreted to imply addition and subtraction. From the responses offered in these tasks, this appears to be a very common strategy used by these students. The reliance on the meaning of key words resulted in many incorrect responses. With these restricted understanding of semantics, deaf students are compelled to rely on interpreting fragments to make sense of that to which they do understand. The complex semantics and lexical density of the word problems along with the lack of redundant and supportive information found in such arithmetic word problems hinders the capacity of deaf students to make sense of the tasks, and hence their capacity to offer correct responses.

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